Code No: 121AB **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year Examinations, September - 2023

MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AME, MIE, PTM)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Using Cayley - Hamilton theorem find inverse for $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$. [2]

b) Define Index and Signature of a canonical form. [3]

c) State Lagrange's mean value theorem. [2]

d) In polar coordinates, if $x=r\cos\theta,\ y=r\sin\theta$ then show that $\frac{\partial(x,y)}{\partial(r,\theta)}=r$. [3]

e) Write the relation between beta and gamma functions. [2]

f) Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of gamma function [3]

g) Write the laws of natural growth and decay in mathematical form. [2]

h) Find the particular integral of $\left(D^2+1\right)y=x^2$. [3]

i) State any two properties of Laplace transform. [2]

j) Find the Laplace transform of $(t-1)^2\,u(t-1)$. [3]

PART - B

(50 Marks)

2.a) Prove that $A=\begin{bmatrix}\frac{1}{2}(1+i) & \frac{1}{2}(-1+i)\\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i)\end{bmatrix}$ is a unitary and find A^{-1}.

b) Find the rank of the matrix $\begin{bmatrix}1 & 3 & 4 & 3\\ 3 & 9 & 2 & 3\\ 1 & 3 & 4 & 1\end{bmatrix}$ by reducing it to the normal form. [5+5]

OR

3.a) Test for consistency and solve $5x+3y+7z=4, 3x+26y+2z=9, 7x+2y+10z=5.$

b) Reduce the quadratic form $2x_1x_2+2x_1x_3-2x_2x_3$ into a sum of squares by an orthogonal transformation and also discuss its nature. [5+5]

4.a) Verify Rolle's theorem for $f(x)=x^2-6x+8$ in $[2,4]$.

b) If $u=3x+2y-z,\ v=x-2y+z$ and $w=x(x+2y-z),$ show that they are functionally related, and find the relation. [5+5]

OR

5.a) Find the maximum and minimum values of $x^3+3xy^2-15x^2-15y^2+72x.$

b) A rectangular box open at top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. [5+5]

6.a) Prove that $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta=\pi/\sqrt{2}.$

b) Express $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx$ as a beta function. [5+5]

OR

7.a) Find by the double integration, the area lying inside the circle $r=a\sin\theta$ and outside the Cardioid $r=a(1-\cos\theta).$

b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{a^2}=1.$ [5+5]

8.a) A body kept in air with temperature 25^0C cools from 140^0C to 80^0C in 20 minutes. Find when the body cools down to $30^0C.$

b) Find the orthogonal trajectories of the family of curves $r^n=a^n\cos n\theta.$ [5+5]

OR

9.a) Solve $(D^2+1)y=\sin x\sin 2x.$

b) If a voltage of $20\cos 5t$ is applied to a series circuit consisting of 10 ohm resistor and 2 henry inductor, determine the current at any time t. [5+5]

10.a) Find the Laplace transform of $e^{-t}\int_0^t \frac{\sin t}{t}\,dt.$

b) Find the $L\{f(t)\}$, where $f(t)=\begin{cases}\cos(t-\pi/3), & if\ t>\pi/3\\ 0, & if\ t<\pi/3\end{cases}.$ [5+5]

OR

11.a) Apply convolution theorem to find the inverse Laplace transforms of $\frac{1}{s(s^2+4)}.$

b) Solve $(D^2+2D+5)y=e^t\sin t,$ given that $y(0)=0,\ y'(0)=1$ using Laplace transforms. [5+5]

---ooOoo---